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## Determining Bishop's parameter $\chi$ based on pore size distribution

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28 **Abstract**

29 Extension of the effective stress concept to unsaturated soils has been a major concern  
30 for decades. Recent studies significantly contributed to the understanding of the  
31 fundamentals behind Bishop's parameter  $\chi$  which is generally used to define the  
32 effective stress for unsaturated soils. Examination of the recently proposed methods  
33 showed that the contribution of suction to effective stress was often overestimated,  
34 especially in high suction range. In this study, considering that soil pores with different  
35 sizes contribute differently to the overall hydro-mechanical behaviour, a new method  
36 to determine Bishop's parameter  $\chi$  is proposed. The key variable used in this method  
37 is the ratio of the change in water volume to the change in macro-pore void volume  
38 due to loading at constant suction. Shear strength data for a weakly expansive clay  
39 were used for validation. A good agreement was obtained between prediction and  
40 measurement, indicating the validity of the proposed method.

41

42 **Keywords:**

43 constitutive relations; fabric/structure of soils; shear strength; partial saturation;  
44 suction

45 **NOTATION**

46	$e_{w2}$	water ratio caused by change of strain at constant suction
47	$e_w^m$	microscopic water ratio
48	$e^m$	microscopic void ratio
49	$S_r^e$	effective degree of saturation
50	$S_r^m$	microscopic degree of saturation
51	$S_r^{cap}$	capillary degree of saturation
52	$S_r^{ads}$	adsorbed degree of saturation
53	$s_m$	suction corresponding to the median pore diameter
54	$V_{w1}$	water volume governed by suction
55	$\alpha$	ratio of the change in incremental water volume caused by the change of
56		strain under constant suction to total pore volume increment
57	$\lambda$	material parameter defining the effective degree of saturation
58	$\xi$	standard deviation of the log-transformed pore radius
59	$\varphi$	fitting parameter related to the theoretical degree of saturation due to
60		adsorption at 1 kPa suction
61	$\chi$	Bishop's effective stress parameter
62		

63 **INTRODUCTION**

64 The definition of stress variables is essential in developing constitutive models for  
65 unsaturated soils. There are two approaches for modeling their mechanical behaviour:  
66 the effective stress approach (Bishop, 1959) and the two independent state variables  
67 approach (Coleman 1962, Fredlund & Morgenstern, 1977). Currently, using two  
68 independent stress variables to describe the behavior of unsaturated soils is widely  
69 accepted (Cui and Delage, 1996; Wheeler *et al.*, 2003; Liu *et al.*, 2018). Bishop stress  
70 is often considered as one variable in the two independent stresses approach, because  
71 it represents a smooth extension of Terzaghi's effective stress (Sheng *et al.*, 2004;  
72 Pereira *et al.*, 2005; Zhou *et al.*, 2018). In the Bishop's effective stress concept, the  
73 key parameter is  $\chi$ , as shown in Eq. 1 (Bishop, 1959):

74 
$$\sigma' = \sigma - [\chi u_w + (1 - \chi)u_a] \quad (1)$$

75 where  $\sigma'$  is unsaturated effective stress;  $\sigma$  is total stress;  $\chi$  is the effective stress  
76 parameter (Bishop's parameter);  $u_w$  and  $u_a$  are water and air pressures, respectively.

77 In the past few decades, various criteria were reported for determining  $\chi$ . In the  
78 early contributions (Bishop & Blight, 1963),  $\chi$  was set equal to degree of saturation  $S_r$ :

79 
$$\chi = S_r \quad (2)$$

80 which is the simplest, and most used assumption to date. But several studies showed  
81 that this choice significantly overestimates the contribution of suction to effective  
82 stress, especially in the high suction range (Sheng *et al.*, 2011; Zhou *et al.*, 2016). In  
83 a later effort, the overestimation was reduced by using an effective degree of  
84 saturation (Vanapalli *et al.*, 1996). To define this latter, attempts have been made to

85 directly relate  $\chi$  to the total degree of saturation, such as (Vanapalli *et al.*, 1996):

$$86 \quad \chi = f(S_r) = (S_r)^\lambda \quad (3)$$

87 where  $\lambda \geq 1$  is a material parameter. For coarse-grained soils, parameter  $\lambda$  is almost  
88 equal to unity, while for fine-grained soils  $\lambda$  is larger than unity. Garven & Vanapalli  
89 (2006) found that  $\lambda$  is correlated with plasticity index,  $I_p$ .

90 Soil water consists in capillary water and adsorbed one. Since adsorbed water is  
91 strongly bonded to soil particles (Lu *et al.*, 2010; Konrad & Lebeau, 2015; Zhou *et*  
92 *al.*, 2016; Gao *et al.*, 2018), it does not contribute to the contact stress and thus to the  
93 effective stress. Thereby, Zhou *et al.* (2016) related the Bishop's parameter  $\chi$  to the  
94 capillary degree of saturation  $S_r^{\text{cap}}$  which is the difference between the total degree  
95 of saturation and the adsorbed degree of saturation  $S_r^{\text{ads}}$ :

$$96 \quad \chi = S_r^{\text{cap}} = \frac{C(s) - \varphi C(s)A(s)}{1 - \varphi C(s)A(s)} \quad (4)$$

$$97 \quad \text{with } C(s) = \frac{1}{2} \operatorname{erfc} \left[ \frac{\ln(s/s_m)}{\sqrt{2}\xi} \right], \quad A(s) = 1 - \frac{\ln(s)}{\ln(s_d)}$$

98 where  $s$  is matric suction;  $\varphi$  and  $\xi$  are fitting parameters determined by soil water  
99 retention curve (SWRC);  $s_m$  is the suction which corresponds to the median pore  
100 diameter;  $s_d$  is the suction at extreme dry state, assumed equal to  $10^6$  kPa. However,  
101 the latest investigation showed that using capillary degree of saturation  
102 underestimates the contribution of suction to Bishop's effective stress for most soils  
103 (Gao *et al.*, 2020). It has been well documented that for compacted soils, the  
104 microstructure is characterised by two pore populations: i) macro-pores where  
105 capillary effects dominate, and ii) micro-pores where the hygroscopic effect  
106 dominates (Romero & Vaunat, 2000; Alonso *et al.*, 2010). It has been admitted that

107 the contribution of suction to Bishop's effective stress is only related to the water  
 108 trapped in macro- or inter-aggregate pores (Alonso *et al.*, 2010; Zhai *et al.*, 2019).  
 109 This was confirmed by some experimental results which showed that loading and  
 110 drying paths predominantly influenced the macropores, while micropores remain  
 111 almost undisturbed (Monroy *et al.*, 2010; Mašín, 2013). Therefore, Bishop's  
 112 parameter  $\chi$  can be considered as the effective degree of saturation of macro-pores  $S_r^e$   
 113 (Alonso *et al.*, 2010):

$$114 \quad \chi = S_r^e = \frac{S_r - S_r^m}{1 - S_r^m} \quad (5)$$

115 where  $S_r^m$  the degree of saturation of micro-pores. This method can improve the  
 116 prediction of effective stress, but still overestimates the contribution of suction in  
 117 some cases (Sheng *et al.*, 2011; Zhai *et al.*, 2019).

118 In this study, a new method for determining  $\chi$  using the macro-PSD is proposed,  
 119 which satisfies Houlsby's power equation.

## 120 **DETERMINING BISHOP'S PARAMETER $\chi$ BASED ON MACROPOROSITY**

121 Fig. 1 shows a conceptual sketch of water retention curves at two different void ratios  
 122 (Vaunat & Casini, 2017). The incremental change in water ratio  $\delta e_w$  (path OB) caused  
 123 either by suction change or mechanical loading can be split into  $\delta e_{w1}$  (path OA) and  
 124  $\delta e_{w2}$  (path AB). The change of water content along path OA is caused by the increment  
 125 of suction at constant volume, while the incremental component of path AB is caused  
 126 by the change of void ratio under constant suction (net stress effect). This latter can  
 127 be expressed as a proportion of total volume change (Vaunat & Casini, 2017). The  
 128 total incremental work input per unit volume in unsaturated soils  $\delta w$  (Houlsby, 1997)

129 can be expressed as:

$$\begin{aligned} 130 \quad \delta w &= -[\sigma - \alpha u_w - (1 - \alpha)u_a] \frac{\delta V_V}{V} - (u_a - u_w) \frac{\delta V_{w1}}{V} \\ 131 \quad &= -[\sigma - \alpha u_w - (1 - \alpha)u_a] \delta \varepsilon_V - (u_a - u_w) \frac{\delta V_{w1}}{V} \end{aligned} \quad (6)$$

132 where  $\sigma$  is the total mean stress;  $V$  is the total soil volume;  $V_V$  is the volume of voids;  
133  $\delta V_{w1}$  is the change of volume occupied by water due to suction;  $\alpha$  is the ratio of the  
134 increment of volume occupied by water caused by the incremental volumetric strain  
135 under constant suction to the pore volume increment. As the first term between  
136 brackets in Eq. (6) expresses Bishop's effective stress, Vaunat & Casini (2017)  
137 proposed:

$$138 \quad \chi = \alpha = \frac{\delta V_{w2}}{\delta V_V} = \frac{\delta e_{w2}}{\delta e} \quad (7)$$

139 where  $\delta e_{w2}$  is the increment of water ratio caused by the change of volumetric strain  
140 under constant suction;  $\delta e$  is the increment of void ratio.

141 Fig. 2 shows the SWRCs under different void ratios for several soils (Li *et al.*,  
142 2007; Salager *et al.*, 2013; Seiphoori *et al.*, 2014; Gao & Sun, 2017). It appears that  
143 in low suction range, the water retention curve is dependent on void ratio. By contrast,  
144 in higher suction range, the SWRCs are almost the same (Fig. 3). The separating point  
145 corresponds to the delimitation between the range of capillary water in macro-pores  
146 and the range of adsorbed water in micro-pores (Romero, 1999). Mařín (2013) also  
147 discussed the structure evolution with mechanical loading and assumed that loading  
148 (compaction) influenced predominantly the macro-pores, while micro-pores remained  
149 untouched (Fig. 3). Thus, as the volume change is mainly due to macro-pores changes,  
150 Bishop's parameter  $\chi$  is solely related to the low suction range (Fig. 3). Thereby, Eq.

151 (7) is modified as:

$$152 \quad \chi = \frac{\delta e_{w2} - \delta e_w^m}{\delta e - \delta e^m} \quad (8)$$

153 where  $\delta e_w^m$  is the incremental microscopic water ratio;  $\delta e^m$  is the incremental  
154 microscopic void ratio. In Fig. 3, two SWRCs at different void ratios should be  
155 provided and the void ratio of every SWRC should be kept constant in the whole  
156 suction range. However, in a SWRC test, the void ratio changes with suction changes.  
157 As the pore size distribution (PSD) obtained by Mercury Intrusion Porosimetry (MIP)  
158 test can be used to determine the SWRC at a constant void ratio (Zhang *et al.*, 2018),  
159 the parameters in Eq. (8) can be obtained using two different PSDs, as illustrated in  
160 Fig. 4.

## 161 **EVALUATION OF THE PROPOSED METHOD THROUGH SHEAR** 162 **STRENGTH**

163 Bishop & Blight (1963) described the shear strength of unsaturated soils by:

$$164 \quad \tau_f = c' + [(\sigma_n - u_a) + \chi(u_a - u_w)] \tan \theta' \quad (9)$$

165 where  $\tau_f$  is the shear strength;  $c'$  is the effective cohesion at saturated state;  $\sigma_n$  is  
166 the normal stress;  $\theta'$  is the internal friction angle. Using Eq. 9 and considering  
167 triaxial conditions, parameter  $\chi$  can be back-calculated, as follows:

$$168 \quad \chi = \frac{\frac{q_f - c'}{3 - \sin \theta'} \frac{6 \cos \theta'}{M} \bar{p}}{s} \quad (10)$$

169 where  $q_f$  is the deviator stress at failure;  $M = 6 \sin \phi' / (3 - \sin \phi')$  is the slope of critical  
170 state line;  $\bar{p} = [\sigma_1 + 2\sigma_3] / 3 - u_a$  is the net mean stress.

171 The results from the tests on Nanyang expansive clay were used to validate the

172 proposed method for Bishop's parameter  $\chi$  determination (Fig. 8). Table 1 summarises  
173 the physical property indexes and triaxial shear parameters for this clay. Triaxial shear  
174 tests were conducted using an unsaturated triaxial testing apparatus supplied by GDS  
175 company. In order to obtain the molded samples only in drying process, all triaxial  
176 specimens were prepared by static compaction at initial water content of about 0.215  
177 (suction is about 170 kPa) and initial dry density around 1.25 Mg/m<sup>3</sup>. Table 2 gives a  
178 summary of the test conditions and stress paths. For tests No.1-3, suction was applied  
179 using axis-translation technique in triaxial cell, while for tests No.4 and No.5 the  
180 suctions (0.8 and 2.5 MPa, respectively ) were applied through controlling the water  
181 contents ( $w=14.8\%$  and  $w=12.55\%$ , respectively) by referring to the SWRC in drying  
182 process. For higher suctions, *i.e.* tests No.6-8, the vapor equilibrium method was  
183 employed. When the samples of tests No.4-8 reached the target respective suctions,  
184 they were put into triaxial cell for further consolidation and shear tests following the  
185 stress and suction paths shown in Figs. 5 (a) and (b). The net confining pressure was  
186 100 kPa, and the suctions were 0.05, 0.2, 0.4, 0.8, 2.5, 3.29, 38 and 368 MPa,  
187 respectively. In the triaxial shear tests,  $q_f$  and  $\bar{p}$  were measured for every suction state  
188 and given in Table 3. As illustrated in Fig. 2, when the suction is higher than a specific  
189 value, the water retention curve is independent of void ratio. In addition, the modulus  
190 of soils with high suction are normally very large and the changes of void during shear  
191 tests are thus small. Thereby, shear tests under constant water content are often  
192 considered as under constant suction in high suction range (Gao *et al.*, 2019; Zhang  
193 *et al.*, 2020). Substitution of  $q_f$ ,  $\bar{p}$ ,  $c'$   $\theta'$  and  $s$  (see Tables 1, 2 and 3) into Eq. (10)

194 allows the calculation of Bishop's parameter  $\chi$  for every suction (or degree of  
195 saturation), as illustrated in Fig. 8.

196 As the void ratio of this sample with 3.29 MPa in shear test is 0.99, the PSD  
197 curves of two samples compacted at different void ratios (0.81 and 0.92) at the same  
198 constant suction of 3.29 MPa are considered. The results are shown in Fig. 6.  
199 According to Lloret *et al.* (2003), the macro- and micro-pores are separated by the  
200 point where the PSD becomes independent of loading. For bimodal structure, the  
201 diameter at the valley bottom between the two pore size families can be considered as  
202 the delimiting diameter between micro- and macro-pores. It appears from Fig. 6 that  
203 the PSD curves are not affected by the compaction load when the pore diameter is  
204 smaller than 4900 nm. Thereby, this diameter was taken as the boundary between  
205 macro- and micro-pores. Eq. (8) can then be applied to determine Bishop's parameter  
206  $\chi$  using the macro-PSDs. Fig. 7 shows the obtained results. For comparison, the  
207 parameters in Table 1 are substituted into Eq. (10), and several other methods are  
208 applied including Bishop's method with  $\chi = S_r$  (Eq. 2), Vanapalli *et al.*'s method (Eq.  
209 3), Zhou *et al.*'s method (Eq. 4), Alonso *et al.*'s method (Eq. 5), and Vaunat & Casini's  
210 method (Eq. 7). Results are included in Fig. 8. It is observed that when the global  
211 degree of saturation (Eq. 2) is considered as Bishop's parameter  $\chi$ , significant  
212 overestimation of suction contribution is obtained, in agreement with the observation  
213 of Alonso *et al.* (2010) and Zhou *et al.* (2016). In Zhou *et al.*'s method, the capillary  
214 water is separated from the adsorbed water. Then the capillary degree of saturation  
215 (Eq. 4) is used to determine Bishop's parameter  $\chi$ . Fig. 7 shows the fitted SWRCs by

216 Zhou *et al.*' SWRC method (Zhou *et al.*, 2016) and the corresponding parameters. The  
217 relationship between capillary degree of saturation and degree of saturation is plotted  
218 in Fig. 8. It appears that this method underestimates the suction contribution to  
219 Bishop's effective stress over a wide suction range. In Alonso *et al.*'s method,  $S_r^m$  is  
220 the degree of saturation of the micro-pores with diameter smaller than 4900 nm. It  
221 was found to be about 20% by calculation based on the PSD in Fig. 6. The effective  
222 degree of saturation (Eq. 5) is used to determine  $\chi$ . It appears that the prediction is  
223 improved, but the suction contribution is overestimated. Similarly, Vaunat & Casini's  
224 method can improve the prediction in the low suction range as Alonso *et al.*'s method,  
225 but remains unsatisfactory in the higher suction range. In particular, a small bump on  
226 the curve obtained by Vaunat & Casini's method appears in the range of degree of  
227 saturation from 5% to 40%, owing to the contribution of micro-pores to the calculation  
228 of  $\chi$ . In Vanapalli *et al.*'s method, a value of 1.6 was fitted for exponent parameter  $\lambda$ .  
229 It is observed that the prediction matches the test data well in the low suction range,  
230 but gives higher  $\chi$  values in higher suction range. Interestingly, the proposed method  
231 (Eq. 8) shows a good agreement with the experimental results in the full suction range.

## 232 CONCLUSIONS

233 The hydro-mechanical behavior of unsaturated soils is strongly related to the water  
234 distribution in pores. As loading and drying predominantly affect the macro-pores,  
235 with the micro-pores remaining almost undisturbed, the pore size below which the  
236 PSD curves become independent of loading or drying processes can be considered as  
237 the delimiting diameter between macro-and micro-pores. This also implies that the

238 contribution of suction to Bishop's effective stress is only related to the water in  
239 macro-pores.

240 Based on a method proposed by Vaunat & Casini (2017) for Bishop's parameter  
241  $\chi$  determination, a modified method was proposed considering the contribution of  
242 capillary water in macro-pores. Bishop's parameter  $\chi$  was defined as the ratio of the  
243 change in -water volume to the change in macro-pores during a loading process at  
244 constant suction. When the degree of saturation of macro-pores is zero,  $\chi$  becomes  
245 zero too. The proposed method was evaluated using experimental shear strength data  
246 and compared with different methods reported in literature. It appeared that the  
247 proposed method allows good agreement between the prediction and measurement  
248 over a wide suction range, as opposed to other methods which either overestimate or  
249 underestimate the suction contribution to the effective stress. It is however worth  
250 noting that more test results are needed to further validate the proposed method.

251

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255

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339 **Table captions.**

340 Table 1 Physical property indexes and shear strength parameters of Nanyang weakly  
341 expansive soil

342 Table 2 Relevant state variables in triaxial tests

343 Table 3 Stress state at triaxial shear failure

344 **Figure captions.**

345 Fig. 1 Partition of total water volume change into components due to suction and  
346 deformation only for a path going from the water retention curve at  $e$  to  $e+\delta e$  (after  
347 Vaunat & Casini, 2017)

348 Fig. 2. SWRCs under different void ratios over a wide suction range for soils: (a) a  
349 clayey silty sand (data from after Salager et al., 2013); (b) MX-80 granular bentonite  
350 (data from Seiphoori et al., 2014); (c) Maryland clay (data from Li et al., 2007); (d)  
351 Pearl clay (data from Gao & Sun, 2017)

352 Fig. 3 Sketches of soil-water retention behavior of specimens with different densities  
353 over a wide suction range

354 Fig. 4 PSDs at two different void ratios (compacted at different void ratios at a  
355 constant suction)

356 Fig. 5 Stress and suction paths for tests: (a) tests in the lower suction range; (b) tests  
357 in the higher suction range

358 Fig. 6 The PSD of Nanyang weakly expansive soil and the criterion adopted to  
359 discriminate macro- and micro-pores

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364

365 **Table 1 Physical property indexes and shear strength parameters of Nanyang weakly expansive**  
 366 **soil**

Specific gravity	Liquid limit (%)	Plastic limit (%)	Plasticity index	Shrinkage limit (%)	Maximum dry density (g/cm <sup>3</sup> )	Optimum water content (%)	Free swelling ratio (%)	Effective cohesion (kPa)	Effective friction angle (°)
2.74	38.8	17.2	21.6	10.5	1.69	18.2	53.8	10.4	20.8

367  
368

369 **Table 2 Relevant state variables in triaxial tests**

Test No.	Molding state			Before triaxial shearing			Suction (MPa)	Control suction method	Net cell pressure (kPa)
	$w_0$	$S_{r0}(\%)$	$e_0$	$w$	$S_r(\%)$	$e$			
1	21.50	48.45	1.216	24.14	79.13	0.826	0.05	Axis-translation technique	100
2	22.01	50.13	1.203	19.12	52.79	0.982	0.20		
3	21.50	48.97	1.203	17.07	45.30	1.02	0.40		
4	21.64	49.54	1.197	14.80	38.44	1.04	0.80	Air-drying	
5	21.78	49.57	1.204	12.55	35.52	0.957	2.50		
6	21.86	49.95	1.199	11.90	32.53	0.996	3.29	Vapor equilibrium technique	
7	21.72	49.51	1.202	6.60	19.57	0.924	38.0		
8	21.76	49.52	1.204	4.10	12.20	0.907	367.5		

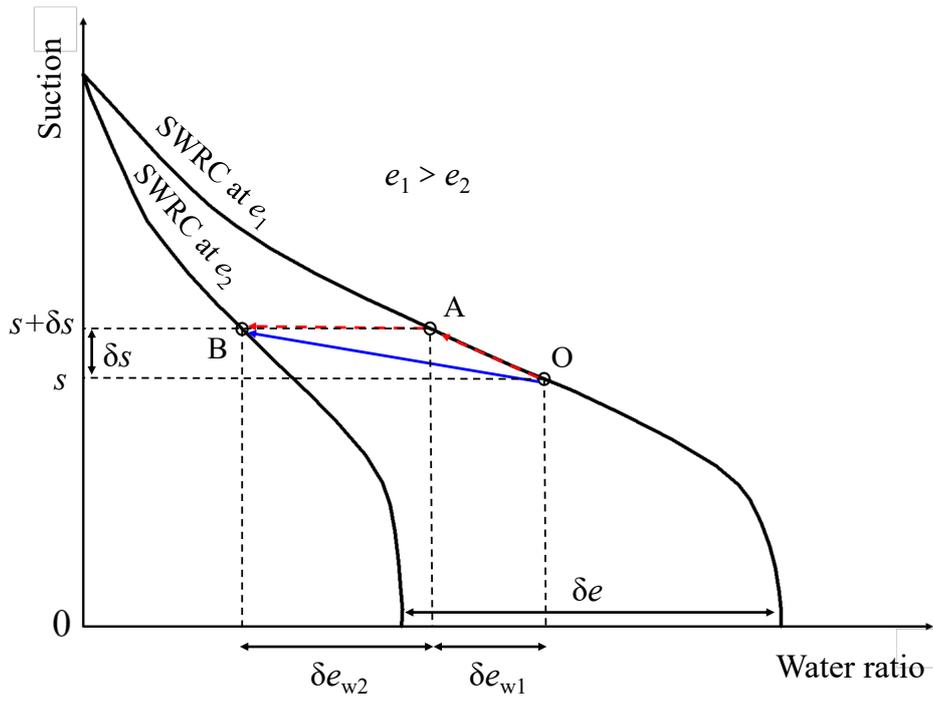
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371 **Table 3 Stress state at triaxial shear failure**

Test No.	Control suction method	Net cell pressure (kPa)	Deviator stress $q_f$ (kPa)	Net mean stress $\bar{p}$ (kPa)
1	Axis-translation technique	100	140	146
2			210	160
3			280	196
4	Air-drying		320	210
5			600	300
6	Vapor equilibrium technique		660	326
7			1290	550
8			1680	660

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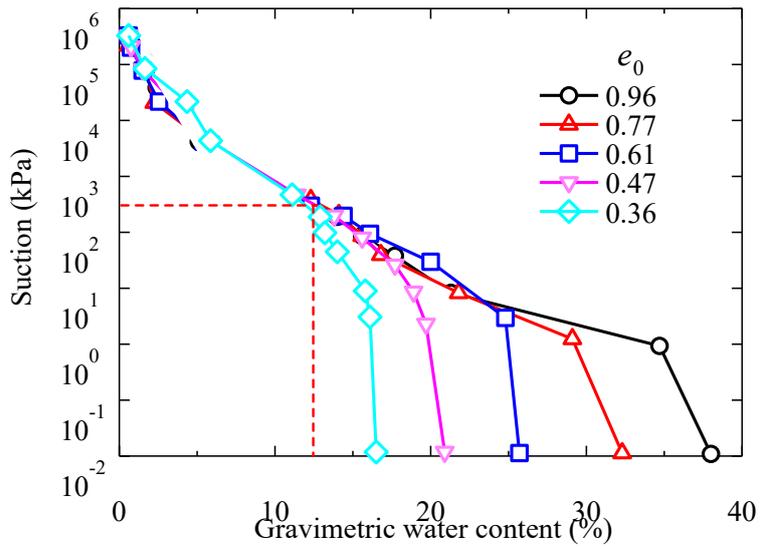
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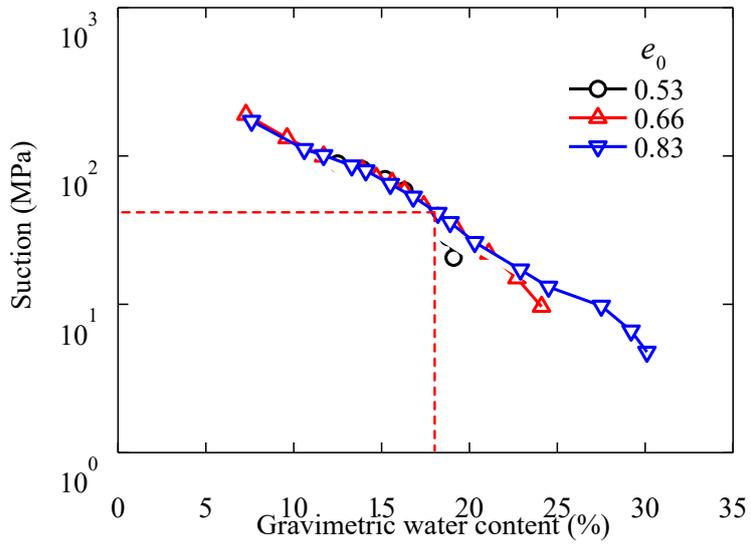
375 Fig. 1. Partition of total water volume change into components due to suction and  
 376 deformation only for a path going from the water retention curve at  $e$  to  $e + \delta e$  (after  
 377 Vaunat & Casini, 2017)

378



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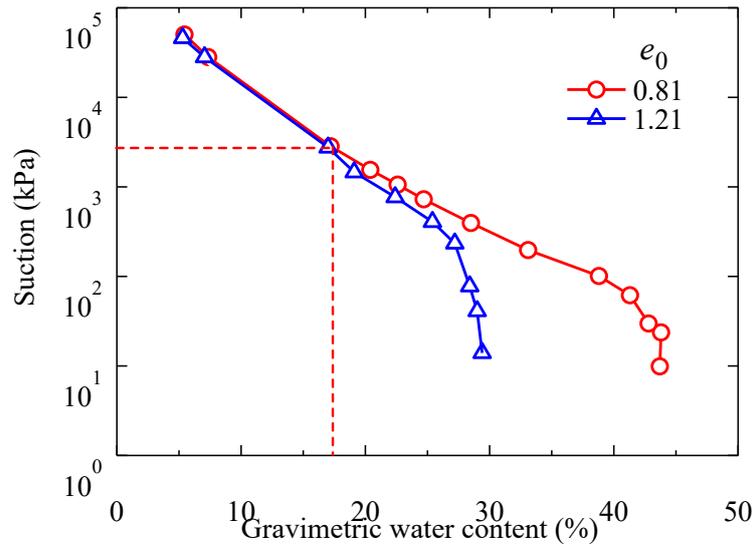
(a)



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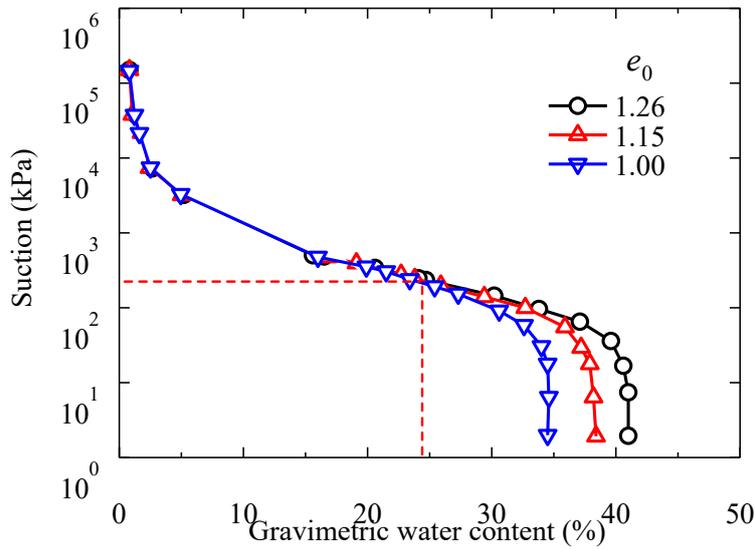
(b)

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(c)

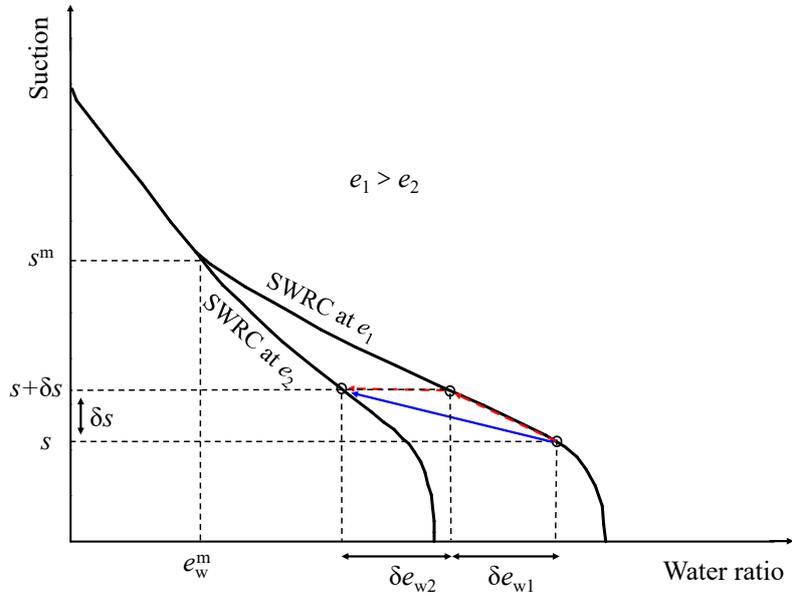
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(d)

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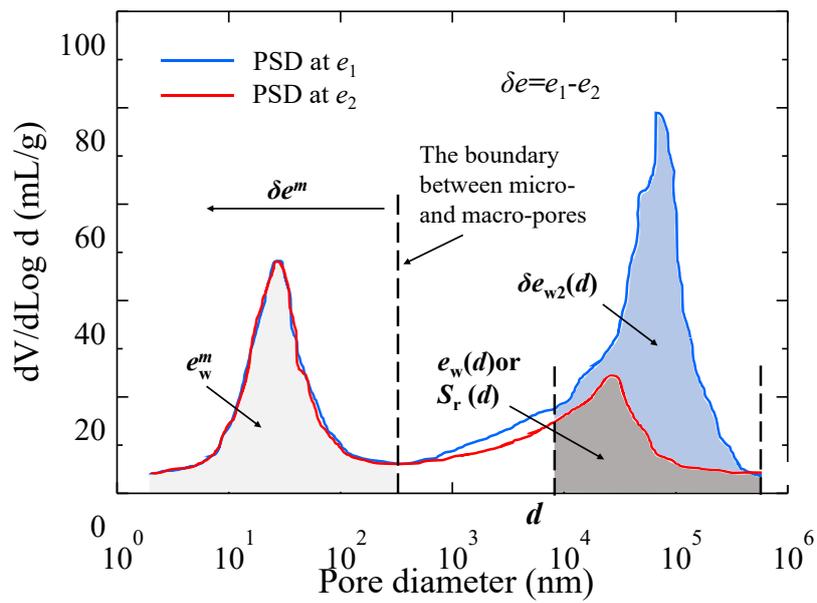
384 Fig. 2. SWRCs under different void ratios over a wide suction range for soils: (a) a  
 385 clayey silty sand (data from after Salager *et al.*, 2013); (b) MX-80 granular bentonite  
 386 (data from Seiphoori *et al.*, 2014); (c) Maryland clay (data from Li *et al.*, 2007); (d)  
 387 Pearl clay (data from Gao & Sun, 2017)



388

389 Fig. 3. Sketches of soil-water retention behavior of specimens with different densities

390 over a wide suction range

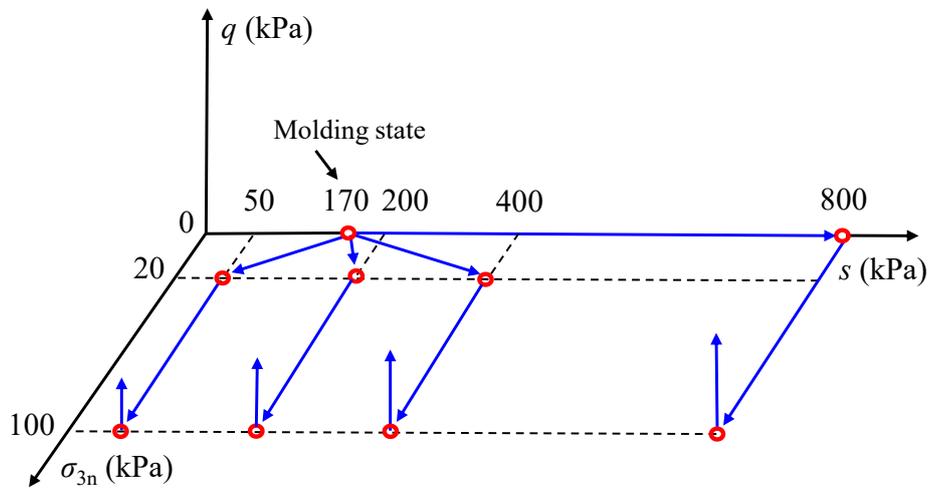


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392 Fig. 4. PSDs at two different void ratios (compacted at different void ratios at a

393 constant suction)

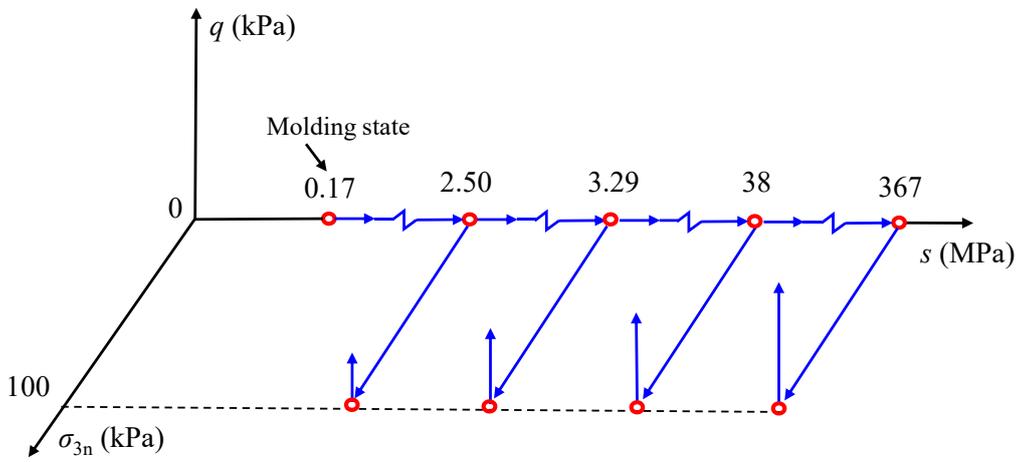
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(a)

395

396



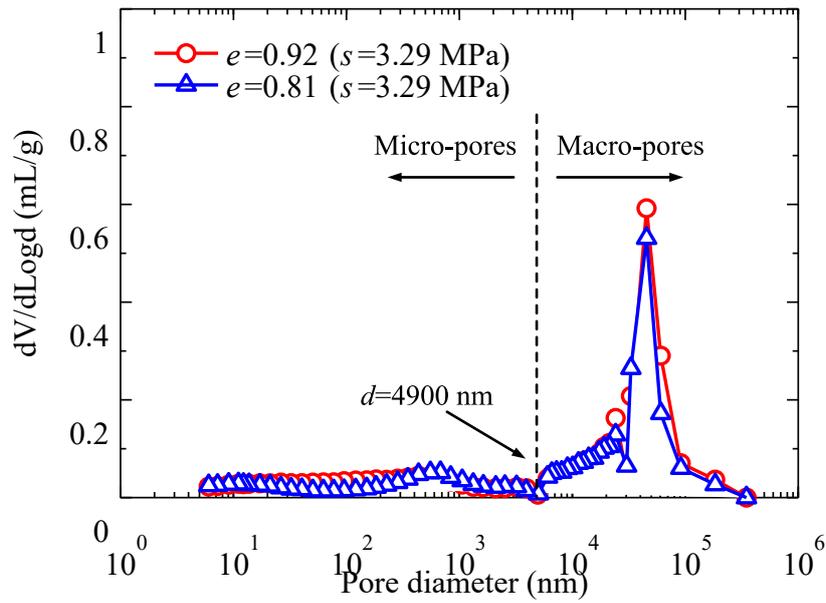
(b)

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398 Fig. 5. Stress and suction paths for tests: (a) tests in the lower suction range; (b) tests

399 in the higher suction range

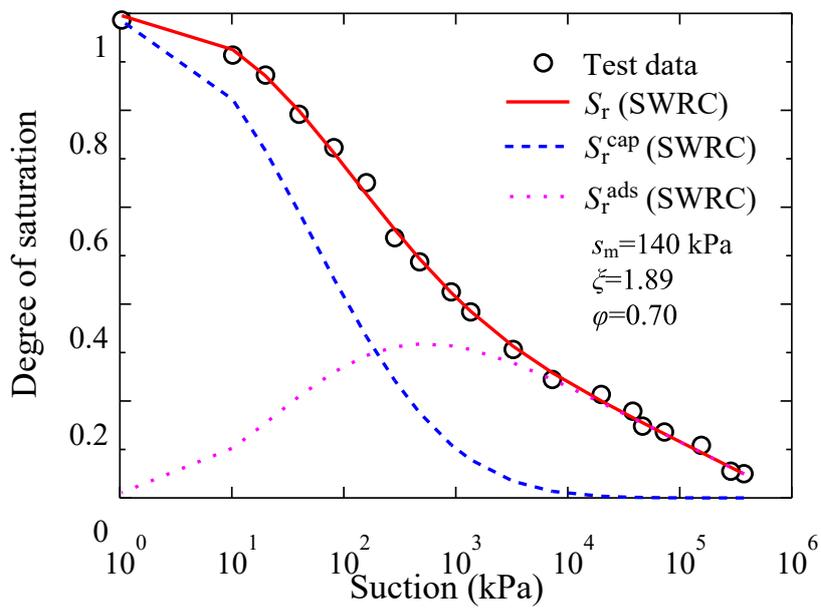
400



401

402 Fig. 6. The PSDs of Nanyang weakly expansive soil and the criterion adopted to

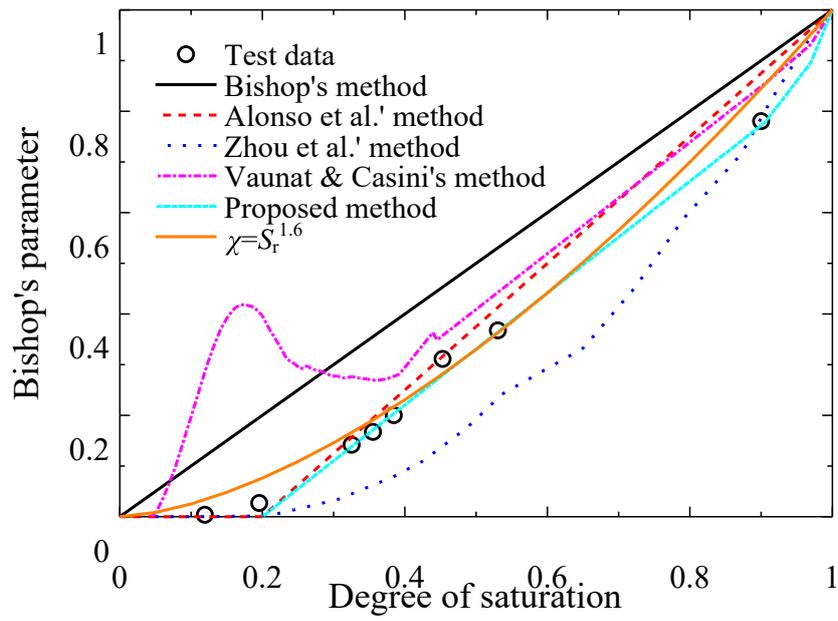
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404

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